Kinematics of Differential Drive Vehicles

CIS 390

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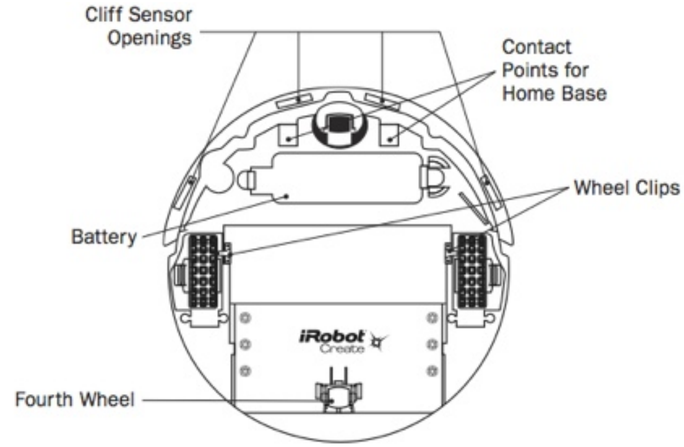
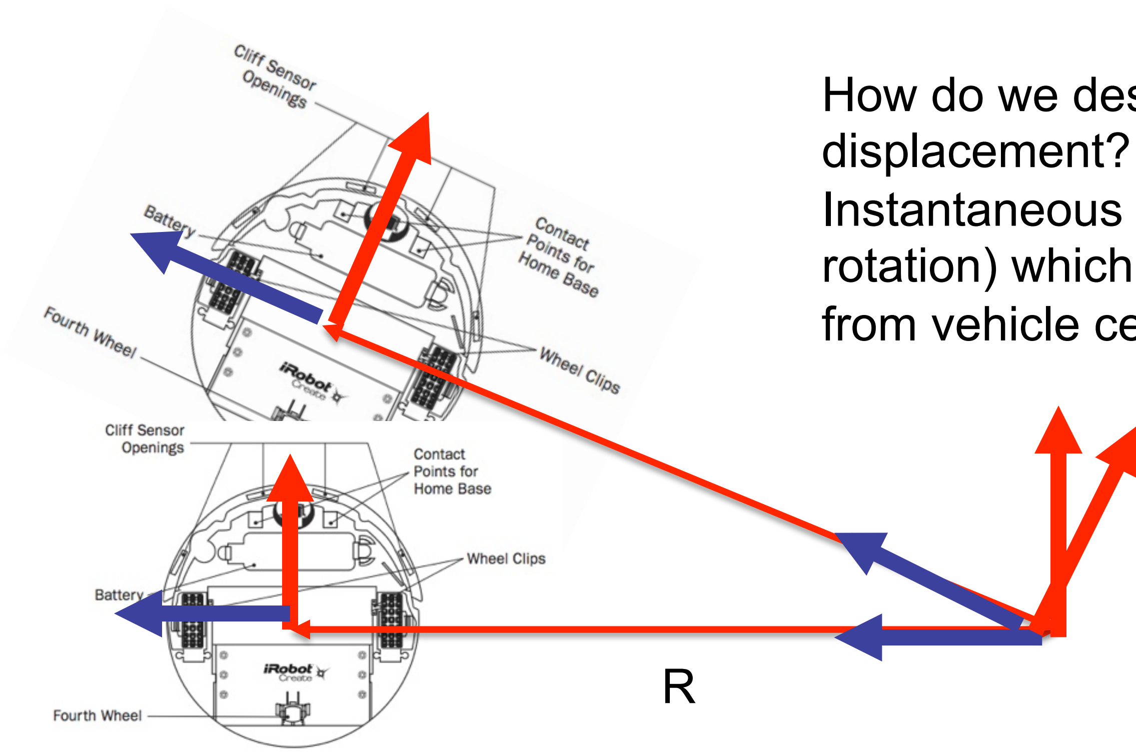
Differential Drive Robots:
iRobot Create



Robots with two active parallel wheels on an axis



How do we describe a very small displacement? With the ICC: Instantaneous center of curvature (or rotation) which is located at radius R from vehicle center.

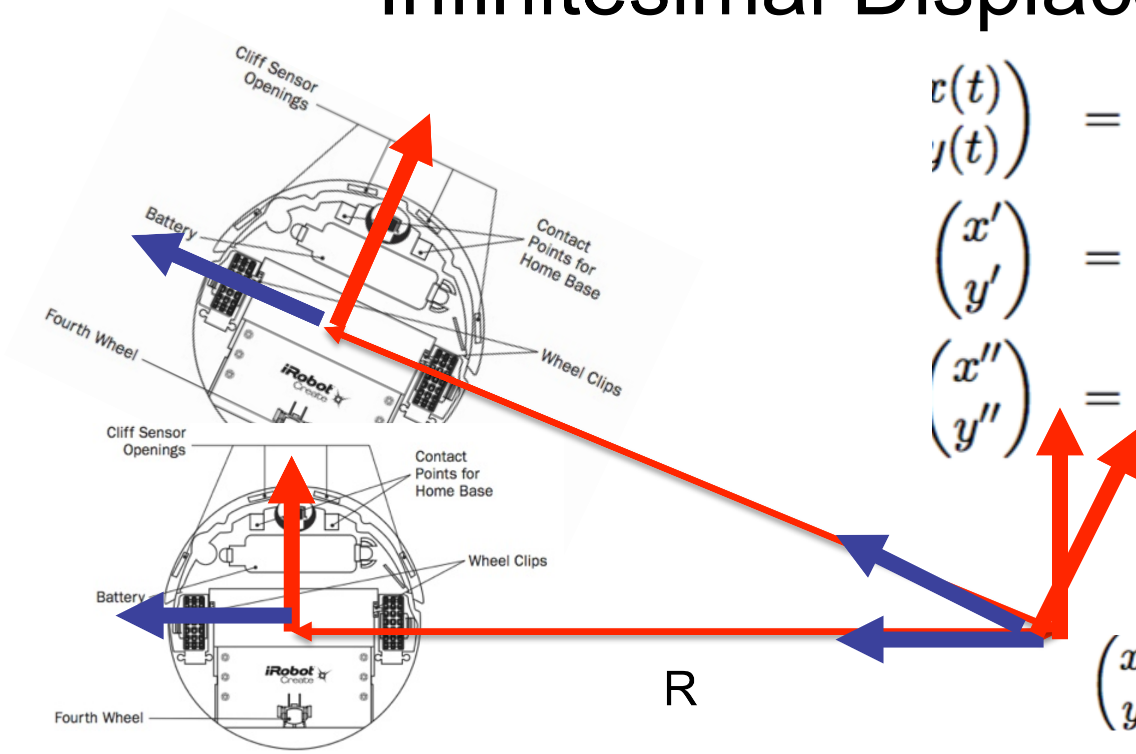
The transformation can be described as a translation parallel to the wheels to ICC along the y-axis followed by a rotation, and a translation back.

Infinitesimal Displacement

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_{ICC} \\ y_{ICC} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos(\omega\Delta t) & \sin(\omega\Delta t) \\ -\sin(\omega\Delta t) & \cos(\omega\Delta t) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} x(t+\Delta t) \\ y(t+\Delta t) \end{pmatrix} - \begin{pmatrix} x_{ICC} \\ y_{ICC} \end{pmatrix}$$



$$\begin{pmatrix} x_{ICC} \\ y_{ICC} \end{pmatrix}$$

Transformation from time t to time t+δt

$$\begin{pmatrix} x(t+\delta t) \\ y(t+\delta t) \end{pmatrix} = \begin{pmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) \\ \sin(\omega\delta t) & \cos(\omega\delta t) \end{pmatrix} \begin{pmatrix} x - x_{ICC} \\ y - y_{ICC} \end{pmatrix} + \begin{pmatrix} x_{ICC} \\ y_{ICC} \end{pmatrix}$$

$$\theta(t+\delta t) = \theta(t) + \omega\delta t$$

We see angular velocity ω in the equation but we do not see here linear velocity (speed) v

$$v = \omega R$$

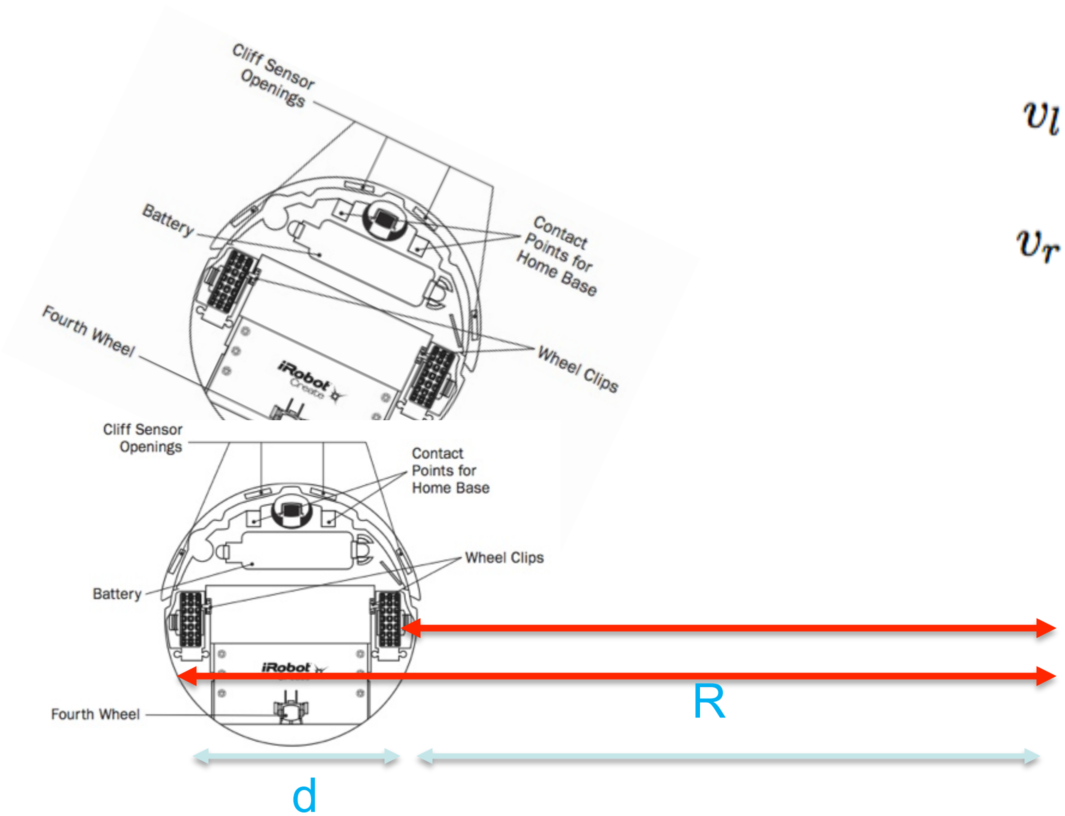
$$\begin{pmatrix} x_{ICC} \\ y_{ICC} \end{pmatrix} = \begin{pmatrix} 0 \\ -R \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{v}{\omega} \end{pmatrix}$$

So, we know where the robot will go if our input command is constant v *and* ω.

But what about if our input is the speeds of the two wheels?

Forward kinematics problem: Given the speeds of the wheels v_r and v_l find x,y,θ after δt.

We need to find R and ω from v_r and v_l. We realize that we need the vehicle width. Apply v=ωr for each wheel. Assume d is the vehicle width.



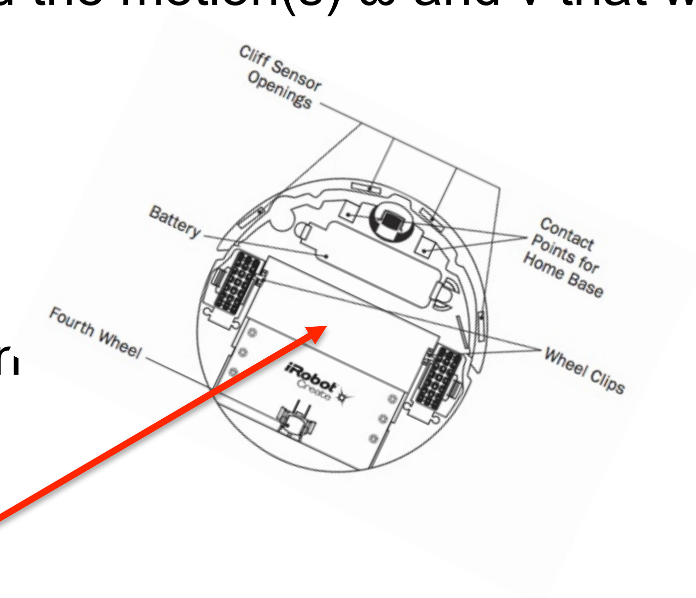
$$\begin{aligned} v_l &= \omega(R + \frac{d}{2}) \\ v_r &= \omega(R - \frac{d}{2}) \end{aligned}$$

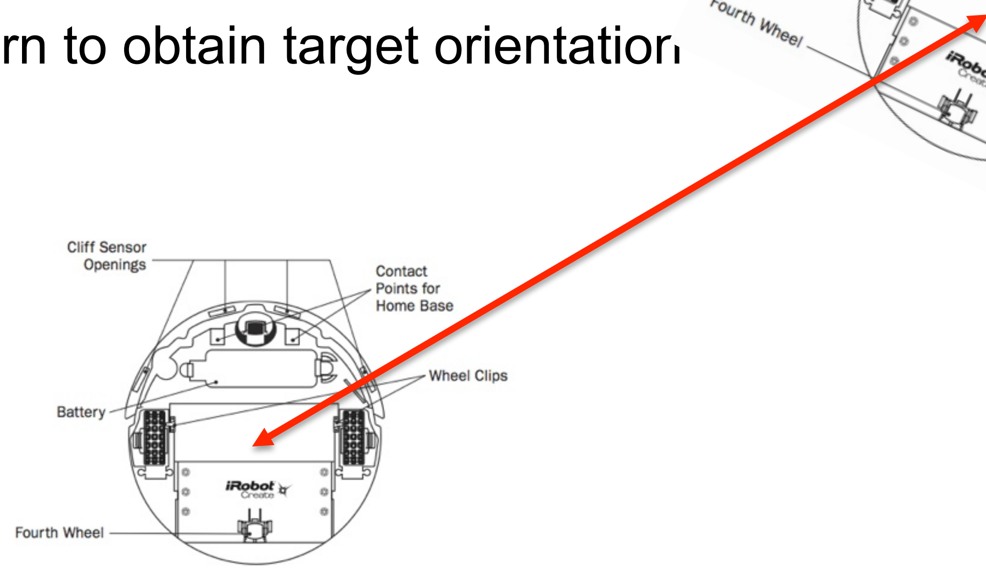
yields

$$\begin{aligned} \omega &= \frac{v_l - v_r}{d} \\ v &= \omega R = \frac{v_l + v_r}{2} \\ R &= \frac{d(v_l + v_r)}{2(v_l - v_r)} \end{aligned}$$

Inverse kinematics problem

- Given a target position (x,y,θ) find the motion(s) ω and v that will lead to this position:
- Naïve solution:
- 1. Turn towards target pose
- 2. Move forward
- 3. Turn to obtain target orientation

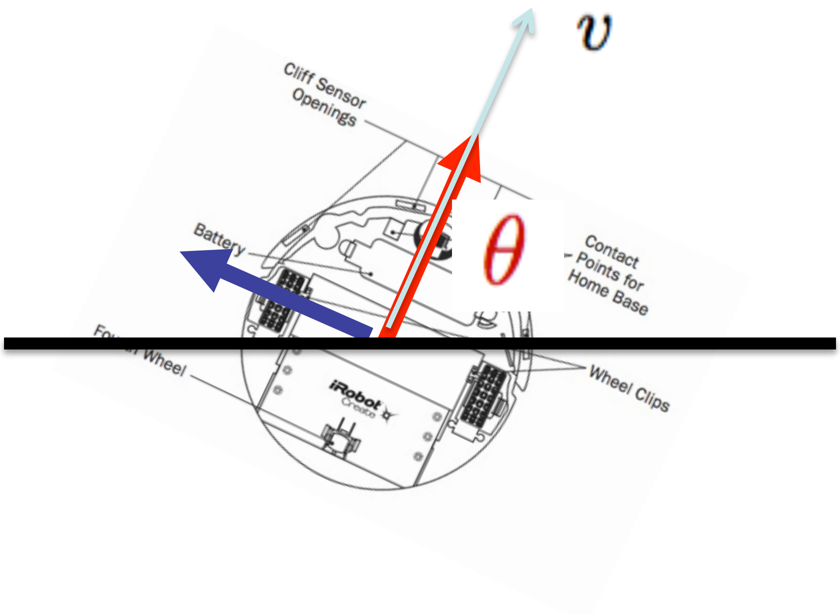




Non-holonomic constraints

- Why is that complicated?
- Because a car can not move sidewards!

- A non-holonomic constraint is a constraint on the feasible ***velocities*** of a body. Linear velocity *v* is parallel to x axis:



$$
\begin{aligned}
\dot{x} &= v \cos\theta \\
\dot{y} &= v \sin\theta \\
\dot{\theta} &= \omega
\end{aligned}
$$

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0$$