Deep Learning
An MIT Press book
Ian Goodfellow and Yoshua Bengio and Aaron Courville

Exercises  Lectures  External Links

The Deep Learning textbook is a resource intended to help students and practitioners enter the field of machine learning in general and deep learning in particular. The online version of the book is now complete and will remain available online for free.

The deep learning textbook can now be pre-ordered on Amazon. Pre-orders should ship on December 16, 2016.

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Citing the book

To cite this book, please use this bibtex entry:

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  year={2016}
}
```
Neural Networks and Deep Learning

Neural Networks and Deep Learning is a free online book. The book will teach you about:

- Neural networks, a beautiful biologically-inspired programming paradigm which enables a computer to learn from observational data
- Deep learning, a powerful set of techniques for learning in neural networks

Neural networks and deep learning currently provide the best solutions to many problems in image recognition, speech recognition, and natural language processing. This book will teach you many of the core concepts behind neural networks and deep learning.

For more details about the approach taken in the book, see here. Or you can jump directly to Chapter 1 and get started.
reviews

» http://www.deeplearningbook.org/contents/linear_algebra.html
» http://www.deeplearningbook.org/contents/prob.html
» http://www.deeplearningbook.org/contents/numerical.html
Grading

» Project + Report: 80%
» Midterm: 20%

» Project:
  – each student pick a topic related to one of the lectures
  – write a literature review
  – presentation in class
  – propose and implement an algorithm
  – Final presentation
Convolutional Neural Networks
Earliest “deep” architecture

Neocognitron

(Fukushima 1974-1982)
**Goal:** Given an image, we want to identify what class that image belongs to.
Pipeline:

Input

Convolutional Neural Network (CNN)

Output

A Monitor
Convolutional Neural Nets (CNNs) in a nutshell:

• A typical CNN takes a raw RGB image as an input.
• It then applies a series of non-linear operations on top of each other.
• These include convolution, sigmoid, matrix multiplication, and pooling (subsampling) operations.
• The output of a CNN is a highly non-linear function of the raw RGB image pixels.
How the key operations are encoded in standard CNNs:

- Convolutional Layers: 2D Convolution
- Fully Connected Layers: Matrix Multiplication
- Sigmoid Layers: Sigmoid function
- Pooling Layers: Subsampling
2D convolution:

\[ h = f \otimes g \]

- \(f\) - the values in a 2D grid that we want to convolve
- \(g\) - convolutional weights of size \(M \times N\)

\[
h_{ij} = \sum_{m=0}^{M} \sum_{n=0}^{N} f(i-m, j-n)g(m,n)\]

A sliding window operation across the entire grid \(f\).
\[ f = \]

\[ g_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0.107 & 0.113 & 0.107 \\ 0.113 & 0.119 & 0.113 \\ 0.107 & 0.113 & 0.107 \end{pmatrix} \quad g_3 = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix} \]

\[ f \odot g_1 \quad f \odot g_2 \quad f \odot g_3 \]

Unchanged Image  
Blurred Image  
Vertical Edges
CNNs aim to learn convolutional weights directly from the data
Early layers learn to detect low level structures such as oriented edges, colors and corners.
Deep layers learn to detect high-level object structures and their parts.
A Closer Look inside the Convolutional Layer

A Chair Filter
A Person Filter
A Table Filter
A Cupboard Filter

Input Image
Fully Connected Layers:

\[ z_i^{(l)} \] - the output unit \( i \) in layer \( l \)

\[ W_{ij}^{(l)} \] - the weight connection between unit \( j \) in layer \( l \) and unit \( i \) in layer \( l + 1 \)

Layer 0

\[ z_1^{(1)} = W_{11}^{(0)} x_1 + W_{12}^{(0)} x_2 + W_{13}^{(0)} x_3 \]

\[ z_2^{(1)} = W_{21}^{(0)} x_1 + W_{22}^{(0)} x_2 + W_{23}^{(0)} x_3 \]

Layer 1

\[ z_3^{(1)} = W_{31}^{(0)} x_1 + W_{32}^{(0)} x_2 + W_{33}^{(0)} x_3 \]
**Fully Connected Layers:**

![Diagram of fully connected layers](image)

- $z_i^{(l)}$ - the output unit $i$ in layer $l$
- $W_{ij}^{(l)}$ - the weight connection between unit $j$ in layer $l$ and unit $i$ in layer $l+1$

\[
\begin{align*}
\bar{z}^{(1)} &= W^{(0)} x \\
\text{matrix multiplication}
\end{align*}
\]
Max Pooling Layer:

- Sliding window is applied on a grid of values.
- The maximum is computed using the values in the current window.

```
  1  2  3
  4  5  6
  7  8  9
```
Max Pooling Layer:

- Sliding window is applied on a grid of values.
- The maximum is computed using the values in the current window.
Max Pooling Layer:

- Sliding window is applied on a grid of values.
- The maximum is computed using the values in the current window.

```
 1 2 3  
 4 5 6  
 7 8 9  
```

```
 5 6  
```

Max Pooling Layer:

- Sliding window is applied on a grid of values.
- The maximum is computed using the values in the current window.

```
1 2 3
4 5 6
7 8 9
```

```
5 6
8 9
```
**Sigmoid Layer:**

- Applies a sigmoid function on an input

\[
a^{(l)} = f(z^{(l)}) = \frac{1}{1 + \exp(-z^{(l)})}
\]
Convolutional Networks

Let us now consider a CNN with a specific architecture:

• 2 convolutional layers.
• 2 pooling layers.
• 2 fully connected layers.
• 3 sigmoid layers.
Notation:
- □ - convolutional layer output
- ■ - fully connected layer output
- ├── - pooling layer
- │  └── - sigmoid function \( f \)
- └── - softmax function

Forward Pass:

\[ x \]
Notation:
- convolutional layer output
- pooling layer
- fully connected layer output
- sigmoid function $f$
- softmax function

Forward Pass:

$\mathbf{x} \rightarrow \hat{z}^{(1)}$
Notation:
- □ - convolutional layer output
- □ - fully connected layer output
- □ - pooling layer
- □ - sigmoid function \( f \)
- □ - softmax function

Forward Pass:

\( x \) → \( z^{(1)} \) → \( a^{(1)} \)
Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$x$ → $z^{(1)}$ → $a^{(1)}$
Notation:

- convolutional layer output
- pooling layer
- fully connected layer output
- sigmoid function $f$
- softmax function

Forward Pass:

$x \rightarrow z^{(1)} \rightarrow a^{(1)} \rightarrow z^{(2)}$
Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$\mathbf{x}$  $\rightarrow$  $z^{(1)}$  $a^{(1)}$  $\rightarrow$  $z^{(2)}$  $a^{(2)}$
Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function \( f \)
- softmax function

Forward Pass:

\[ x \rightarrow z^{(1)} \rightarrow a^{(1)} \rightarrow z^{(2)} \rightarrow a^{(2)} \]
Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$\mathbf{x} \rightarrow \mathbf{z}^{(1)} \rightarrow \mathbf{a}^{(1)} \rightarrow \mathbf{z}^{(2)} \rightarrow \mathbf{a}^{(2)} \rightarrow \mathbf{z}^{(3)}$
Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$x \xrightarrow{} z^{(1)} a^{(1)} \xrightarrow{} z^{(2)} a^{(2)} \xrightarrow{} z^{(3)} a^{(3)}$
**Convolutional Networks**

**Notation:**
- □ - convolutional layer output
- □ - fully connected layer output
- ┊ - pooling layer
- ┊ - sigmoid function \( f \)
- ┊ - softmax function

**Forward Pass:**

\[ x \rightarrow z^{(1)} \rightarrow a^{(1)} \rightarrow z^{(2)} \rightarrow a^{(2)} \rightarrow z^{(3)} \rightarrow a^{(3)} \rightarrow z^{(4)} \]
Notation:

- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$x \rightarrow z^{(1)} a^{(1)} \rightarrow z^{(2)} a^{(2)} \rightarrow z^{(3)} a^{(3)} \rightarrow z^{(4)} \hat{y}$
Notation:

- convolutional layer output
- pooling layer
- fully connected layer output
- sigmoid function $f$
- softmax function

Forward Pass:

$\mathbf{x} \rightarrow \mathbf{z}^{(1)} \rightarrow \mathbf{a}^{(1)} \rightarrow \mathbf{z}^{(2)} \rightarrow \mathbf{a}^{(2)} \rightarrow \mathbf{z}^{(3)} \rightarrow \mathbf{a}^{(3)} \rightarrow \mathbf{z}^{(4)} \rightarrow \hat{\mathbf{y}}$

Final Predictions
**Notation:**

- □ - convolutional layer output
- ■ - fully connected layer output
- - pooling layer
- - sigmoid function $f$
- - softmax function

**Forward Pass:**

$\mathbf{x}$ $\rightarrow$ $g^{(1)}$ $\rightarrow$ $z^{(1)}$ $\rightarrow$ $a^{(1)}$ $\rightarrow$ $z^{(2)}$ $\rightarrow$ $a^{(2)}$ $\rightarrow$ $z^{(3)}$ $\rightarrow$ $a^{(3)}$ $\rightarrow$ $z^{(4)}$ $\hat{y}$

Convolutional layer parameters in layers 1 and 2
Notation:
- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$x \rightarrow g^{(1)} \rightarrow z^{(1)} a^{(1)} \rightarrow g^{(2)} \rightarrow z^{(2)} a^{(2)} \rightarrow z^{(3)} a^{(3)} \rightarrow z^{(4)} \hat{y}$

Fully connected layer parameters in the fully connected layers 1 and 2
Notation:
- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

$x \rightarrow g^{(1)} \rightarrow z^{(1)} \rightarrow a^{(1)} \rightarrow g^{(2)} \rightarrow z^{(2)} \rightarrow a^{(2)} \rightarrow W^{(1)} \rightarrow z^{(3)} \rightarrow a^{(3)} \rightarrow W^{(2)} \rightarrow z^{(4)} \hat{y}$
Notation:
- convolutional layer output
- pooling layer
- fully connected layer output
- sigmoid function $f$
- softmax function

Forward Pass:

1. $a^{(1)} = pool(f(g^{(1)} \ast x))$
Notation:
- convolutional layer output
- fully connected layer output
- pooling layer
- sigmoid function $f$
- softmax function

Forward Pass:

1. $a^{(1)} = \text{pool}(f(g^{(1)} \ast x))$
2. $a^{(2)} = \text{pool}(f(g^{(2)} \ast a^{(1)}))$
Notation:

- conv= convolutional layer output
- fully_connected= fully connected layer output
- pool= pooling layer
- sigm= sigmoid function $f$
- softmax= softmax function

Forward Pass:

$x \rightarrow g^{(1)} \rightarrow z^{(1)} \rightarrow a^{(1)} \rightarrow g^{(2)} \rightarrow z^{(2)} \rightarrow a^{(2)} \rightarrow W^{(1)} \rightarrow z^{(3)} \rightarrow a^{(3)} \rightarrow W^{(2)} \rightarrow z^{(4)} \hat{y}$

1. $a^{(1)} = \text{pool}(f(g^{(1)} \ast x))$
2. $a^{(2)} = \text{pool}(f(g^{(2)} \ast a^{(1)}))$
3. $a^{(3)} = f(W^{(1)}a^{(2)})$
Forward Pass:

1. $a^{(1)} = \text{pool}(f(g^{(1)} \ast x))$
2. $a^{(2)} = \text{pool}(f(g^{(2)} \ast a^{(1)}))$
3. $a^{(3)} = f(W^{(1)}a^{(2)})$
4. $\hat{y} = \text{softmax}(W^{(2)}a^{(3)})$

Notation:
- box - convolutional layer output
- vertical bar - fully connected layer output
- square - pooling layer
- vertical double bars - sigmoid function $f$
- vertical single bars - softmax function
Key Question: How to learn the parameters from the data?
Backpropagation
for
Convolutional Neural Networks
How to learn the parameters of a CNN?

- Assume that we are given a labeled training dataset
  \[ \{(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})\} \]

- We want to adjust the parameters of a CNN such that CNN’s predictions would be as close to true labels as possible.

- This is difficult to do because the learning objective is highly non-linear.
Gradient descent:

- Iteratively minimizes the objective function.
- The function needs to be differentiable.

\[ \theta = \theta - \alpha \frac{\partial L(\theta)}{\partial \theta} \]
Gradient descent:
- Iteratively minimizes the objective function.
- The function needs to be differentiable.

$$\theta = \theta - \alpha \frac{\partial L(\theta)}{\partial \theta}$$
Gradient descent:

- Iteratively minimizes the objective function.
- The function needs to be differentiable.

\[ \theta = \theta - \alpha \frac{\partial L(\theta)}{\partial \theta} \]
Gradient descent:

• Iteratively minimizes the objective function.
• The function needs to be differentiable.

\[ \theta = \theta - \alpha \frac{\partial L(\theta)}{\partial \theta} \]
1. Compute the gradients of the overall loss w.r.t. to our predictions and propagate it back: 

\[ \frac{\partial L}{\partial \hat{y}} \]
2. Compute the gradients of the overall loss and propagate it back: \[
\frac{\partial L}{\partial z^{(4)}}
\]
3. Compute the gradients to adjust the weights:

\[ \frac{\partial L}{\partial W^{(2)}} \]
4. Backpropagate the gradients to previous layers: \[ \frac{\partial L}{\partial z^{(3)}} \]
5. Compute the gradients to adjust the weights: \[ \frac{\partial L}{\partial W^{(1)}} \]
6. Backpropagate the gradients to previous layers:

\[
\frac{\partial L}{\partial z^{(2)}}
\]
Compute the gradients to adjust the weights:

\[ \frac{\partial L}{\partial g^{(2)}} \]
8. Backpropagate the gradients to previous layers:

\[ \frac{\partial L}{\partial z^{(1)}} \]
9. Compute the gradients to adjust the weights:

\[
\frac{\partial L}{\partial g^{(1)}}
\]
An Example of Backpropagation Convolutional Neural Networks
Assume that we have $K=5$ object classes:

- **Class 1**: Penguin
- **Class 2**: Building
- **Class 3**: Chair
- **Class 4**: Person
- **Class 5**: Bird

\[
\hat{y} = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}
\]

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
$L = - \sum_{i=1}^{K} y_i \log(\hat{y}_i)$ where $\hat{y}_i = \frac{\exp(z_i^{(4)})}{\sum_{j=1}^{K} \exp(z_j^{(4)})}$
\[ L = - \sum_{i=1}^{K} y_i \log(\hat{y}_i) \quad \text{where} \quad \hat{y}_i = \frac{\exp(z_i^{(4)})}{\sum_{j=1}^{K} \exp(z_j^{(4)})} \]

\[ \frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z_i^{(4)}} \]
\[ L = - \sum_{i=1}^{K} y_i \log (\hat{y}_i) \quad \text{where} \quad \hat{y}_i = \frac{\exp(z_i^{(4)})}{\sum_{j=1}^{K} \exp(z_j^{(4)})} \]

\[
\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}_i} \cdot \frac{\partial \hat{y}_i}{\partial z_i^{(4)}}
\]

\[
\frac{\partial L}{\partial \hat{y}_i} = -\frac{y_i}{\hat{y}_i}
\]
\[ L = - \sum_{i=1}^{K} y_i \log (\hat{y}_i) \quad \text{where} \quad \hat{y}_i = \frac{\exp (z_i^{(4)})}{\sum_{j=1}^{K} \exp (z_j^{(4)})} \]

\[
\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}_i} \cdot \frac{\partial \hat{y}_i}{\partial z_i^{(4)}}
\]

\[
\frac{\partial L}{\partial \hat{y}_i} = -y_i / \hat{y}_i
\]

\[
\frac{\partial \hat{y}_i}{\partial z_j^{(4)}} = \begin{cases} 
\hat{y}_i (1 - \hat{y}_i), & \text{if} \quad i = j \\
-\hat{y}_i \hat{y}_j, & \text{if} \quad i \neq j
\end{cases}
\]
\[
\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z_i^{(4)}}
\]
\[
\frac{\partial L}{\partial z^{(4)}_i} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z^{(4)}_i} \\
= \frac{\partial L}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial z^{(4)}_i} + \sum_{i \neq j} \frac{\partial L}{\partial \hat{y}_j} \frac{\partial \hat{y}_j}{\partial z^{(4)}_i}
\]
\[
\frac{\partial L}{\partial z_i^{(4)}} = \frac{\partial L}{\partial \hat{y}_i} \frac{\partial \hat{y}}{\partial z_i^{(4)}} \\
= \frac{\partial L}{\partial \hat{y}_i} \frac{\partial \hat{y}_i}{\partial z_i^{(4)}} + \sum_{i \neq j} \frac{\partial L}{\partial \hat{y}_j} \frac{\partial \hat{y}_j}{\partial z_i^{(4)}} \\
= \hat{y}_i - y_i
\]
Assume that we have K=5 object classes:

- **Class 1:** Penguin
- **Class 2:** Building
- **Class 3:** Chair
- **Class 4:** Person
- **Class 5:** Bird

\[
\hat{y} = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}
\]

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]
Assume that we have $K=5$ object classes:

Class 1: Penguin
Class 2: Building
Class 3: Chair
Class 4: Person
Class 5: Bird

\[
\hat{y} = \begin{bmatrix}
0.5 & 0 & 0.1 & 0.2 & 0.1
\end{bmatrix}
\]

\[
y = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
Assume that we have K=5 object classes:

Class 1: **Penguin**
Class 2: **Building**
Class 3: **Chair**
Class 4: **Person**
Class 5: **Bird**

\[
\hat{y} = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}
\]

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
\frac{\partial L}{\partial z^{(4)}} = \begin{bmatrix} -0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix}
\]

Increasing the score corresponding to the true class decreases the loss.
Assume that we have K=5 object classes:

Class 1: Penguin
Class 2: Building
Class 3: Chair
Class 4: Person
Class 5: Bird

\[ \hat{y} = \begin{bmatrix} 0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix} \]

\[ y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ \frac{\partial L}{\partial z^{(4)}} = \begin{bmatrix} -0.5 & 0 & 0.1 & 0.2 & 0.1 \end{bmatrix} \]

Decreasing the score of other classes also decreases the loss.
Adjusting the weights:
Adjusting the weights:

Need to compute the following gradient:
\[
\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}
\]
Adjusting the weights:

Need to compute the following gradient

$$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}
\]

\[
\frac{\partial L}{\partial z^{(4)}}
\]

was already computed in the previous step
Adjusting the weights:

Need to compute the following gradient

$$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$
Adjusting the weights:

Need to compute the following gradient

$$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}$$

$$z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)})$$

where $f(z^{(3)}) = a^{(3)}$
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}
\]

\[
z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)}) \quad \text{where} \quad f(z^{(3)}) = a^{(3)}
\]

\[
\frac{\partial z_i^{(4)}}{\partial W_{ij}^{(2)}} = f(z_j^{(3)})
\]
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial W^{(2)}}
\]

Update rule:

\[
W_{ij}^{(2)} = W_{ij}^{(2)} - \alpha \frac{\partial L}{\partial W_{ij}^{(2)}}
\]
Backpropagating the gradients:
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial y} \frac{\partial f(y)}{\partial y}
\]
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}
\]
Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$

$$\frac{\partial L}{\partial z^{(4)}}$$ was already computed in the previous step.
Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}$$
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}
\]

\[
z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)}) \quad \text{where} \quad f(z^{(3)}) = a^{(3)}
\]
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}
\]

\[
z_i^{(4)} = \sum_{k=1}^{N} W_{ik}^{(2)} f(z_k^{(3)}) \quad \text{where} \quad f(z^{(3)}) = a^{(3)}
\]

\[
\frac{\partial z_i^{(4)}}{\partial f(z_j^{(3)})} = W_{ij}^{(2)}
\]
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \cdot \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \cdot \frac{\partial f(z^{(3)})}{\partial z^{(3)}}
\]
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}
\]

\[
f(z^{(3)}) = \frac{1}{1 + \exp(-z^{(3)})}
\]
Backpropagating the gradients:

We need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(3)}} = \frac{\partial L}{\partial z^{(4)}} \frac{\partial z^{(4)}}{\partial f(z^{(3)})} \frac{\partial f(z^{(3)})}{\partial z^{(3)}}
\]

The activation function is:

\[
f(z^{(3)}) = \frac{1}{1 + \exp(-z^{(3)})}
\]

The derivative of the activation function is:

\[
\frac{\partial f(z^{(3)})}{\partial z^{(3)}} = f(z^{(3)})(1 - f(z^{(3)}))
\]
Adjusting the weights:
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial W^{(1)}} = \frac{\partial L}{\partial z^{(3)}} \frac{\partial z^{(3)}}{\partial W^{(1)}}
\]

Update rule:

\[
W_{ij}^{(1)} = W_{ij}^{(1)} - \alpha \frac{\partial L}{\partial W_{ij}^{(1)}}
\]
Backpropagating the gradients:
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(2)}} = \frac{\partial L}{\partial z^{(3)}} \frac{\partial z^{(3)}}{\partial f(z^{(2)})} \frac{\partial f(z^{(2)})}{\partial z^{(2)}}
\]
Adjusting the weights:
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}}
\]
Adjusting the weights:

Need to compute the following gradient

\[ \frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial a^{(2)}} \frac{\partial a^{(2)}}{\partial g^{(2)}} \]
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}}
\]

\[
\frac{\partial L}{\partial z^{(2)}}
\]

was already computed in the previous step
Adjusting the weights:

Need to compute the following gradient

\[ \frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}} \]
Adjusting the weights:

Need to compute the following gradient

\[
\frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}}
\]

\[
z_{ij}^{(2)} = \sum_{u=0}^{M} \sum_{v=0}^{N} g_{uv}^{(2)} a_{(i-u)(j-v)}^{(1)}
\]

where

\[
f(z^{(1)}) = a^{(1)}
\]
Adjusting the weights:

Need to compute the following gradient

\[ \frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}} \]

where \( f(z^{(1)}) = a^{(1)} \)

\[ \frac{\partial z_{ij}^{(2)}}{\partial g^{(2)}_{mn}} = a^{(1)}_{(i-m)(j-n)} \]
Adjusting the weights:

Need to compute the following gradient

\[ \frac{\partial L}{\partial g^{(2)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial g^{(2)}} \]

Update rule:

\[ g^{(2)}_{mn} = g^{(2)}_{mn} - \alpha \frac{\partial L}{\partial g^{(2)}_{mn}} \]
Backpropagating the gradients:
Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \cdot \frac{\partial f(z^{(1)})}{\partial z^{(1)}}
\]
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \cdot \frac{\partial f(z^{(1)})}{\partial z^{(1)}}
\]

\[\frac{\partial L}{\partial z^{(2)}}\] was already computed in the previous step
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}
\]
Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$

$$z^{(2)}_{ij} = \sum_{u=0}^{M} \sum_{v=0}^{N} g^{(2)}_{uv} a^{(1)}_{i-u}(j-v) \text{ where } f(z^{(1)}) = a^{(1)}$$
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}
\]

\[
z^{(2)}_{ij} = \sum_{u=0}^{M} \sum_{v=0}^{N} g^{(2)}_{uv} a^{(1)}_{(i-u)(j-v)} \text{ where } f(z^{(1)}) = a^{(1)}
\]

\[
\frac{\partial z^{(2)}_{ij}}{\partial a^{(1)}_{(i-m)(j-n)}} = g^{(2)}_{mn}
\]
Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \cdot \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \cdot \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$
Backpropagating the gradients:

Need to compute the following gradient:

\[
\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial z^{(1)}} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}
\]

\[
f(z^{(1)}) = \frac{1}{1 + \exp(-z^{(1)})}
\]
Backpropagating the gradients:

Need to compute the following gradient:

$$\frac{\partial L}{\partial z^{(1)}} = \frac{\partial L}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial f(z^{(1)})} \frac{\partial f(z^{(1)})}{\partial z^{(1)}}$$

$$f(z^{(1)}) = \frac{1}{1 + \exp(-z^{(1)})}$$

$$\frac{\partial f(z^{(1)})}{\partial z^{(1)}} = f(z^{(1)})(1 - f(z^{(1)}))$$
Adjusting the weights:
**Adjusting the weights:**

Need to compute the following gradient

\[
\frac{\partial L}{\partial g^{(1)}} = \frac{\partial L}{\partial z^{(1)}} \frac{\partial z^{(1)}}{\partial g^{(1)}}
\]

Update rule:

\[
g_{mn}^{(1)} = g_{mn}^{(1)} - \alpha \frac{\partial L}{\partial g_{mn}^{(1)}}
\]
Visual illustration
Backpropagation
Convolutional Neural Networks
Fully Connected Layers:

Forward:

\[
W^{(l)} f(z^{(l)}) \Rightarrow z^{(l+1)}
\]
**Fully Connected Layers:**

**Forward:**

\[ \begin{align*}
W^{(l)} & f(z^{(l)}) \\
& z^{(l+1)}
\end{align*} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]

The weight that is used in conjunction with the activation unit of interest

The output

The weight that is used in conjunction with the activation unit of interest
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) \Rightarrow z^{(l+1)} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) \Rightarrow z^{(l+1)} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]
Backpropagation

Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) \Rightarrow z^{(l+1)} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} \ f(z^{(l)}) \ z^{(l+1)} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} \cdot f(z^{(l)}) = z^{(l+1)} \]
**Fully Connected Layers:**

**Forward:**

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]

**Backward:**

\[ \frac{\partial L}{\partial z^{(l+1)}} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]

Backward:

A measure how much an activation unit contributed to the loss

\[ \frac{\partial L}{\partial z^{(l+1)}} = \frac{\partial L}{\partial f(z^{(l)})} \]
Fully Connected Layers:

Forward:

\[ W^{(l)} \cdot f(z^{(l)}) = z^{(l+1)} \]

Backward:

\[ (W^{(l)})^T \cdot \frac{\partial L}{\partial z^{(l+1)}} = \frac{\partial L}{\partial f(z^{(l)})} \]

A measure how much an activation unit contributed to the loss.
Fully Connected Layers:

Forward:

\[ W^{(l)} \cdot f(z^{(l)}) \cdot z^{(l+1)} = \]

Backward:

\[ (W^{(l)})^T \cdot \frac{\partial L}{\partial z^{(l+1)}} \cdot \frac{\partial L}{\partial f(z^{(l)})} = \]
Fully Connected Layers:

Forward:

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]

Backward:

\[ (W^{(l)})^T \frac{\partial L}{\partial z^{(l+1)}} \frac{\partial L}{\partial f(z^{(l)})} \]
Fully Connected Layers:

**Forward:**

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]

**Backward:**

\[ (W^{(l)})^T \frac{\partial L}{\partial z^{(l+1)}} = \frac{\partial L}{\partial f(z^{(l)})} \]
**Fully Connected Layers:**

**Forward:**

\[
W^{(l)} \ f(z^{(l)}) = z^{(l+1)}
\]

**Backward:**

\[
(W^{(l)})^T \ \frac{\partial L}{\partial z^{(l+1)}} \ \frac{\partial L}{\partial f(z^{(l)})}
\]
**Fully Connected Layers:**

**Forward:**

\[ W^{(l)} f(z^{(l)}) z^{(l+1)} = \]

**Backward:**

\[ (W^{(l)})^T \frac{\partial L}{\partial z^{(l+1)}} \frac{\partial L}{\partial f(z^{(l)})} = \]
**Fully Connected Layers:**

**Forward:**

\[ W^{(l)} f(z^{(l)}) = z^{(l+1)} \]

**Backward:**

\[ (W^{(l)})^T \frac{\partial L}{\partial z^{(l+1)}} = \frac{\partial L}{\partial f(z^{(l)})} \]
Summary for fully connected layers

Backpropagation
Convolutional Neural Networks
Summary:

1. Let \( \frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i \), where \( n \) denotes the number of layers in the network.
Summary:

1. Let \( \frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i \), where \( n \) denotes the number of layers in the network.

2. For each fully connected layer \( l \):
   - For each node \( i \) in layer \( l \) set:
     \[
     \frac{\partial L}{\partial z_i^{(l)}} = \left( \sum_{j=1}^{s^{l+1}} W_{ji}^{(l)} \frac{\partial L}{\partial z_j^{(l+1)}} \right) \frac{\partial f(z_i^{(l)})}{\partial z_i^{(l)}}
     \]
Summary:

1. Let \( \frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i \), where n denotes the number of layers in the network.

2. For each fully connected layer \( l \):
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     \frac{\partial L}{\partial z_i^{(l)}} = \left( \sum_{j=1}^{s^{l+1}} W_{ji}^{(l)} \frac{\partial L}{\partial z_j^{(l+1)}} \right) \frac{\partial f(z_i^{(l)})}{\partial z_i^{(l)}}
     \]

   - Compute partial derivatives:

     \[
     \frac{\partial L}{\partial W_{ij}^{(l)}} = f(z_j^{(l)}) \frac{\partial L}{\partial z_i^{(l+1)}}
     \]
Summary:

1. Let \( \frac{\partial L}{\partial z_i^{(n)}} = \hat{y}_i - y_i \), where \( n \) denotes the number of layers in the network.

2. For each fully connected layer \( l \):
   
   • For each node \( i \) in layer \( l \) set:
     \[
     \frac{\partial L}{\partial z_i^{(l)}} = \left( \sum_{j=1}^{s_l+1} W_{ji}^{(l)} \frac{\partial L}{\partial z_j^{(l+1)}} \right) \frac{\partial f(z_i^{(l)})}{\partial z_i^{(l)}}
     \]
   
   • Compute partial derivatives:
     \[
     \frac{\partial L}{\partial W_{ij}^{(l)}} = f(z_j^{(l)}) \frac{\partial L}{\partial z_i^{(l+1)}}
     \]
   
   • Update the parameters:
     \[
     W_{ij}^{(l)} = W_{ij}^{(l)} - \alpha \frac{\partial L}{\partial W_{ij}^{(l)}}
     \]
Convolutional Layers:

Forward:

\[ a^{(l)} \otimes g^{(l)} = z^{(l+1)} \]
Convolutional Layers:

Forward:

\( a^{(l)} \otimes g^{(l)} = z^{(l+1)} \)

Backward:

A measure how much an activation unit contributed to the loss

\[ \frac{\partial L}{\partial z^{(l+1)}} \]

\[ \frac{\partial L}{\partial f(z^{(l)})} \]
**Convolutional Layers:**

**Forward:**

\[ a^{(l)} \otimes g^{(l)} = z^{(l+1)} \]

**Backward:**

\[ \text{sum} \left ( g^{(l)} \odot \frac{\partial L}{\partial z^{(l+1)}} \right ) = \frac{\partial L}{\partial f(z^{(l)})} \]
Summary:

1. Let \( \frac{\partial L}{\partial z^{(c)}} \), where \( c \) denotes the index of a first fully connected layer.

2. For each convolutional layer \( l \):
   - For each node \( i,j \) in layer \( l \) set
     \[
     \frac{\partial L}{\partial z_{i,j}^{(l)}} = \left( \sum_{m=0}^{M} \sum_{n=0}^{N} g_{mn}^{(l)} \frac{\partial L}{\partial z^{(l+1)}_{(i+m)(j+n)}} \right) \frac{\partial f(z_{i,j}^{(l)})}{\partial z_{i,j}^{(l)}}
     \]
Summary:

1. Let $\frac{\partial L}{\partial z^{(c)}}$, where $c$ denotes the index of a first fully connected layer.

2. For each convolutional layer $l$:
   
   • For each node $ij$ in layer $l$ set
     \[
     \frac{\partial L}{\partial z_{ij}^{(l)}} = \left( \sum_{m=0}^{M} \sum_{n=0}^{N} g_{mn}^{(l)} \frac{\partial L}{\partial z_{(i+m)(j+n)}^{(l+1)}} \right) \frac{\partial f(z_{ij}^{(l)})}{\partial z_{ij}^{(l)}}
     \]
   
   • Compute partial derivatives:
     \[
     \frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})
     \]
1. Let \( \frac{\partial L}{\partial z^{(c)}} \), where \( c \) denotes the index of a first fully connected layer.

2. For each convolutional layer \( l \):
   
   • For each node \( ij \) in layer \( l \) set
     
     \[
     \frac{\partial L}{\partial z_{ij}^{(l)}} = \left( \sum_{m=0}^{M} \sum_{n=0}^{N} g_{mn}^{(l)} \frac{\partial L}{\partial z^{(l+1)}_{(i+m)(j+n)}} \right) \frac{\partial f(z_{ij}^{(l)})}{\partial z_{ij}^{(l)}}
     \]

   • Compute partial derivatives:
     
     \[
     \frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})
     \]

   • Update the parameters:
     
     \[
     g_{ij}^{(l)} = g_{ij}^{(l)} - \alpha \frac{\partial L}{\partial g_{ij}^{(l)}}
     \]
Gradient in pooling layers:

- There is no learning done in the pooling layers.
- The error that is backpropagated to the pooling layer, is sent back from to the node where it came from.
Gradient in pooling layers:

- There is no learning done in the pooling layers.
- The error that is backpropagated to the pooling layer, is sent back from to the node where it came from.
Gradient in pooling layers:

- There is no learning done in the pooling layers.
- The error that is backpropagated to the pooling layer, is sent back from the node where it came from.
Recognition as a big table lookup

\[
\begin{array}{c|c}
\text{Image} & \text{Recognition Label} \\
\hline
\text{n} & \text{Y} \\
\text{m} & \text{N} \\
\end{array}
\]

write down all images in a table and record the object label

\[n \times m\]
“How many images are there?”

An tiny image example (can you see what it is?)

Each pixel has $2^8 = 256$ values

3x18 image above has 54 pixels

Total possible 54 pixel images = $256^{54} = 1.1 \times 10^{130}$

Prof. Kanade’s Theorem: we have not seen anything yet!
Total possible 54 pixel images = $1.1 \times 10^{130}$

Compared

<table>
<thead>
<tr>
<th>Total population</th>
<th>years</th>
<th>hours</th>
<th>frame rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 billion</td>
<td>$1000$</td>
<td>$356$</td>
<td>$24$</td>
</tr>
<tr>
<td></td>
<td>$24$</td>
<td>$60$</td>
<td>$30$</td>
</tr>
</tbody>
</table>

We have to be clever in writing down this table!
A Closer Look inside the Convolutional Layer

A Chair Filter

A Person Filter

A Table Filter

A Cupboard Filter

Input Image
A Closer Look inside the Convolutional Layer:

\[ x \xrightarrow{g^{(1)}} z^{(1)} \xrightarrow{a^{(1)}} z^{(2)} \xrightarrow{g^{(2)}} z^{(3)} \xrightarrow{W^{(2)}} \hat{y} \]

\[ y \]
A Closer Look inside the Back Propagation Convolutional Layer:

\[
\frac{\partial L}{\partial z_{ij}^{(l)}} = \left( \sum_{m=0}^{M} \sum_{n=0}^{N} g_{mn}^{(l)} \frac{\partial L}{\partial z_{(i+m)(j+n)}^{(l+1)}} \right) \frac{\partial f(z_{ij}^{(l)})}{\partial z_{ij}^{(l)}}
\]
Adjusting the weights:
Training Batch

\[
\frac{\partial L}{\partial g^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{y,x}^{(l+1)}} f(z_{y,x}^{(l)})
\]

\[
\begin{align*}
\sum (4) \odot & = \begin{array}{c}
\frac{\partial L}{\partial g^{(1)}}^{(1)} \\
\frac{\partial L}{\partial g^{(1)}}^{(2)} \\
\frac{\partial L}{\partial g^{(1)}}^{(3)} \\
\frac{\partial L}{\partial g^{(1)}}^{(4)} \\
\frac{\partial L}{\partial g^{(1)}}^{(5)} 
\end{array} \\
\text{(performed in a sliding window fashion)}
\end{align*}
\]

Average the Gradient Across the Batch

Parameter Update

\[
g^{(1)} = g^{(1)} - \frac{\partial L}{\partial g^{(1)}}
\]

\[
\text{new } g^{(1)} \quad \text{old } g^{(1)} - \frac{\partial L}{\partial g^{(1)}}
\]

\(\odot\) - elementwise multiplication
Training Batch

\[
\frac{\partial L}{\partial g^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z^{(l+1)}_{y,x}} f(z^{(l)}_{y-i,x-j})
\]

\(\sum (4) \odot \sum (1) \odot \sum (2) \odot \sum (3) \odot \sum (5) = \sum (1) \odot \sum (1)
\]

(performing in a sliding window fashion)

\(\bigodot\) - elementwise multiplication

Average the Gradient Across the Batch

Parameter Update

\[g^{(1)} = g^{(1)} - \frac{\partial L}{\partial g^{(1)}}\]
Training Batch

\[
\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})
\]

\[
\sum (\begin{array}{c}
4 \\
1 \\
0 \\
8 \\
1
\end{array}) = \begin{array}{c}
\frac{\partial L}{\partial g^{(1)}}^{(1)} \\
\frac{\partial L}{\partial g^{(1)}}^{(2)} \\
\frac{\partial L}{\partial g^{(1)}}^{(3)} \\
\frac{\partial L}{\partial g^{(1)}}^{(4)} \\
\frac{\partial L}{\partial g^{(1)}}^{(5)}
\end{array}
\]

Average the Gradient Across the Batch

Parameter Update

\[
g^{(1)} = g^{(1)} - \frac{\partial L}{\partial g^{(1)}}
\]

(performed in a sliding window fashion)

\(\odot\) - elementwise multiplication
Adjusting the weights:
Training Batch

\[
\frac{\partial L}{\partial g^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{yx}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})
\]

\[
sum (a^{(1)} \odot \frac{\partial L}{\partial z^{(2)}}) = \sum_{j} \frac{\partial L}{\partial g^{(2)}}
\]

(performing elementwise multiplication)

Average the Gradient Across the Batch

Parameter Update

\[
g^{(2)} = g^{(2)} - \frac{\partial L}{\partial g^{(2)}}
\]

\[
\begin{array}{c}
\text{new } g^{(2)} \\
\text{old } g^{(2)}
\end{array} = \begin{array}{c}
\text{new } g^{(2)} \\
\text{old } g^{(2)}
\end{array} - \frac{\partial L}{\partial g^{(2)}}
\]
Training Batch

\[
\frac{\partial L}{\partial g^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{y,x}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})
\]

\[
\sum (\bullet) \quad \frac{\partial L}{\partial g^{(2)}} (1)
\]

\[
\sum (\bullet) \quad \frac{\partial L}{\partial g^{(2)}} (2)
\]

\[
\sum (\bullet) \quad \frac{\partial L}{\partial g^{(2)}} (3)
\]

\[
\sum (\bullet) \quad \frac{\partial L}{\partial g^{(2)}} (4)
\]

\[
\sum (\bullet) \quad \frac{\partial L}{\partial g^{(2)}} (5)
\]

(performed in a sliding window fashion)

\(\circlearrowright\) - elementwise multiplication

Average the Gradient Across the Batch

Parameter Update

\[
g^{(2)} = g^{(2)} - \frac{\partial L}{\partial g^{(2)}}
\]

new \(g^{(2)}\) = old \(g^{(2)}\) - updated \(g^{(2)}\)
Training Batch

\[
\frac{\partial L}{\partial g_{ij}^{(l)}} = \sum_{y=0}^{H} \sum_{x=0}^{W} \frac{\partial L}{\partial z_{y,x}^{(l+1)}} f(z_{(y-i)(x-j)}^{(l)})
\]

\( \sum ( \circ ) = \frac{\partial L}{\partial g_{ij}^{(2)}} \)

(performed in a sliding window fashion)

\( \sum ( \circ ) = \frac{\partial L}{\partial g_{ij}^{(2)}} \)

\( \sum ( \circ ) = \frac{\partial L}{\partial g_{ij}^{(2)}} \)

\( \sum ( \circ ) = \frac{\partial L}{\partial g_{ij}^{(2)}} \)

\( \sum ( \circ ) = \frac{\partial L}{\partial g_{ij}^{(2)}} \)

\( g^{(2)} = g^{(2)} - \frac{\partial L}{\partial g^{(2)}} \)

\( \text{new } g^{(2)} = \text{old } g^{(2)} - \frac{\partial L}{\partial g^{(2)}} \)

\( \circ \) - elementwise multiplication