Multi-Image Matching via Fast Alternating Minimization
Supplementary Material

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Abstract

This document presents the supplementary material for the ICCV'2015 paper: Multi-Image Matching via Fast Alternating Minimization.

Solving Step 6 in Algorithm 1

We need to solve the following problem:

$$\min_{\boldsymbol{X}\in\mathcal{C}} \|\boldsymbol{X} - \boldsymbol{A}\boldsymbol{B}^T + \frac{1}{\mu}(\boldsymbol{W}+\boldsymbol{Y})\|_F^2, \tag{1}$$

where $\mathcal{C}$ is defined as the set of matrices satisfying:

$$\boldsymbol{X}_{ii} = \boldsymbol{I}_{p_i}, \quad 1 \le i \le n, \tag{2}$$
$$\boldsymbol{X}_{ij} = \boldsymbol{X}_{ji}^T, \ \ 1 \le i, j \le n, i \ne j, \tag{3}$$
$$\boldsymbol{0} \le \boldsymbol{X} \le \boldsymbol{1}, \tag{4}$$

Denoting $\boldsymbol{A}\boldsymbol{B}^T + \frac{1}{\mu}(\boldsymbol{W}+\boldsymbol{Y})$ by $\boldsymbol{Z}$, one can easily verify that each block $\boldsymbol{X}_{ij}$ can be solved separately and the solution is:

$$\begin{aligned} \hat{\boldsymbol{X}}_{ij} &= \boldsymbol{I}_{p_i}, && i = j,\\ \hat{\boldsymbol{X}}_{ij} &= \min\left(\max\left(\frac{1}{2}\left(\boldsymbol{Z}_{ij} + \boldsymbol{Z}_{ji}^T\right), \boldsymbol{0}\right), \boldsymbol{1}\right), && i \ne j, \end{aligned} \tag{5}$$

where $\min$ and $\max$ denote elementwise minimum and maximum operators, respectively.

In Section 5.4 we replace the constraint in (2) as

$$\begin{aligned} \text{trace}(\boldsymbol{X}) &= m',\\ \text{off-diagnal values}\{\boldsymbol{X}_{ii}\} &= \boldsymbol{0}, \ 1 \le i \le n, \end{aligned} \tag{6}$$

The new constraint doesn't affect the solution for blocks where $i \ne j$. For $i = j$, the solution is

$$\begin{aligned} \text{diagnal values}\{\hat{\boldsymbol{X}}\} &= \hat{\boldsymbol{x}},\\ \text{off-diagnal values}\{\hat{\boldsymbol{X}}_{ii}\} &= \boldsymbol{0}, \ 1 \le i \le n, \end{aligned} \tag{7}$$

where $\hat{\boldsymbol{x}}$ is

$$\begin{aligned} \hat{\boldsymbol{x}} &= \arg\min_{\boldsymbol{x}} \|\boldsymbol{x} - \boldsymbol{z}\|^2\\ \text{s.t.} &\ \ \boldsymbol{x}^T\boldsymbol{1} = m', \ \ \boldsymbol{0} \le \boldsymbol{x} \le \boldsymbol{1}, \end{aligned} \tag{8}$$

and $\boldsymbol{z}$ is the vector of diagonal values in $\boldsymbol{Z}$.

Time complexity of Algorithm 1

We analyze the computational cost in each step of Algorithm 1. In Step 4, $\left(\boldsymbol{B}^T\boldsymbol{B} + \frac{\lambda}{\mu}\boldsymbol{I}\right)^\dagger$ requires $O(mk^2)$ and $O(k^3)$ flops for multiplication and inverse, respectively, and the remaining two matrix multiplication steps require $O(mk^2 + m^2k)$ flops. Step 5 has the same complexity with Step 4. In step 6, the dominant cost is to compute $\boldsymbol{A}\boldsymbol{B}^T$ which requires $O(m^2k)$ flops. Step 7 takes negligible time since $\boldsymbol{A}\boldsymbol{B}^T$ has been computed. The total cost of each iteration is $O(3m^2k + 4mk^2 + 2k^3)$ and approximately $O(m^2k)$ when $m > k$.